MTH 1311 – Mathematics for Business Analysis

Hello and Welcome to the weekly resources for MTH 1311 - Mathematics for Business Analysis!

This is Week 10 of class, and typically at this point in the semester, your professors are covering the topics listed below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester. If you have any questions about these study guides, group tutoring sessions, private 30-minute tutoring appointments, the Baylor Tutoring YouTube channel, or any tutoring services we offer, please visit our website at www.baylor.edu/tutoring or call our drop-in center during open business hours: M–Th 9am–7pm on class days at 254-710-4135.

Key Words: Vertex Formula, Maximum/Minimum, Rational Function, Vertical Asymptote, Horizontal Asymptote

Topic of the Week: Quadratic Optimization & Rational Curves (Sections D.4 & D.6)

Highlight 1: Optimizing via the Quadratic Vertex

When a quadratic function ($y = ax^2 + bx + c$) is applied to business models like tracking corporate revenue or operational profit, finding the turning point of the parabola is vital. If the leading coefficient a is negative, the parabola opens downward like a frown, meaning the turning point at the peak is a **maximum point**. If a is positive, the parabola opens upward like a cup, meaning the turning point at the bottom is a **minimum point**.

This turning point is called the **vertex**. Rather than performing complex calculations, you can find the exact input location (x) that optimizes your business model by using this reliable vertex formula:

$$h = \frac{-b}{2a}$$

To determine the actual maximum or minimum financial output value itself (the y-value), simply take your result for h and plug it back into the original function equation to compute f(h).

Highlight 2: Investigating Rational Asymptotes

Rational functions are structured as fractions containing a polynomial expression in the numerator and another polynomial expression in the denominator. Because you cannot divide by zero, these graphs break apart near boundary thresholds called **asymptotes**:

- **Vertical Asymptotes:** These are vertical invisible walls that the graph can never cross or touch. To find them, simplify the fraction completely, set the remaining denominator polynomial equal to zero, and solve for x.
- **Horizontal Asymptotes:** These represent long-run flattening behaviors that show what happens to the output as production sizes grow toward infinity ($x \to \infty$). To locate them, look at the highest degrees (exponents) of your numerator and denominator.
    1. If the denominator degree is larger, the horizontal asymptote is automatically **y = 0**.
    2. If the numerator and denominator degrees are identical, the horizontal asymptote is found by dividing the leading coefficients:

$$y = \frac{\text{leading coefficient of top}}{\text{leading coefficient of bottom}}.$$

In a commercial context, horizontal asymptotes often represent a minimum possible unit production cost that a company can approach but never structurally fall below due to fixed overhead fees.

Things Students Might Struggle With

Confusing the Optimal Input with the Optimal Output: When an exam problem asks *"How many units must be sold to maximize profit?"*, it wants the horizontal vertex input value. If the problem asks *"What is the maximum profit?"*, it wants the vertical output value **f**(h). Pay close attention to the wording ensures you don't write down the wrong half of your vertex pair!

Forgetting to Simplify Before Finding Vertical Asymptotes: A very common mistake is finding a vertical asymptote where a hole in the graph actually exists. You must completely factor both the numerator and denominator and cancel out any shared common factors *before* setting the denominator to zero!

Check Your Learning Workspace

Problem 1: Quadratic Applications and Rational Boundaries

A manufacturing corporation models its daily operational profit using the function

P(x) = -x² + 80x - 700, where x represents the number of units manufactured each day. Separately, their average production cost per unit follows the rational function:

$$C(x) = \frac{15x+600}{x+5}.$$

Part A: Calculate the exact number of units the corporation must produce each day to achieve its absolute maximum profit, and calculate what that maximum profit is.

Part B: Identify the vertical asymptote and the horizontal asymptote for the average cost function C(x).

Closing Comments

The vertex formula is a highly efficient shortcut for optimization. Use it whenever you see a quadratic function and keywords like "maximum profit" or "minimum cost." When transitioning to rational functions, always remember that horizontal asymptotes provide invaluable insight into a company's long-term baseline operational costs!

Answers to the Check Your Learning Problems

Problem 1 Answers:

- **Part A: Identify your quadratic coefficients from P(x) = -x² + 80x - 700: a = -1, b = 80, c = -700. Use the vertex formula to calculate the maximizing input size:**

$$h = \frac{-b}{2a} = \frac{-80}{2(-1)} = \frac{-80}{-2} = 40 \text{ units}$$

Now, evaluate the function at x = 40 to find the absolute maximum profit value:

$$P(40) = -(40)^2 + 80(40) - 700 = -1600 + 3200 - 700 = 900$$

Final Answer: The corporation must produce 40 units to reach a maximum profit of $900.

Part B: For the rational cost function

Vertical Asymptote: Set the simplified denominator equal to zero and solve:

$$x + 5 = 0 \rightarrow x = -5$$

Horizontal Asymptote: Compare the degree of the numerator polynomial against the degree of the denominator polynomial. Both the top ($15x^1$) and bottom (x^1) have an identical highest degree of 1. Divide their leading coefficients:

$$y = \frac{15}{1} \rightarrow y = 15$$

Final Answer: Vertical Asymptote at x = -5; Horizontal Asymptote at y = 15 *(This horizontal line means that as production scales up, the average unit cost approaches a minimum baseline of $15).*