

Week 12

MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is **Week 12 of class**, and typically in this week of the semester your professors are covering these **topics below**. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

KEYWORDS: *U-substitution, Rules of U-substitution, Finding C*

TOPIC OF THE WEEK:

U-Substitution

This week as we progress further into integration, we are covering the methods of integrating. An important topic is u-substitution. U-substitution is a method that is similar to the inverse chain rule of derivatives.

First you must pick a u from your integrated equation. This is usually seen as the greater exponent or the function that is “in a function.”

Once you pick your u, (example: $u = x + 3$) you must differentiate it. The result will give you $(du/dx) = \underline{\hspace{2cm}}$. From the result, you can solve for dx. Once you have it in terms of $dx = \underline{\hspace{2cm}}$, you can substitute that in for dx (see image on right where they substituted $du/g'(x)$ for dx/.

The result allows you to integrate in terms of u and then substitute your value of x back in for u once integrated.

$$\int f(g(x))g'(x)dx$$

$$\int f(u) \cancel{g'(x)} \frac{du}{\cancel{g'(x)}}$$

$$\int f(u)du$$

THIS IS USUALLY SEEN

if $u = g(x)$
 $du = g'(x)dx$
then, $dx = \frac{du}{g'(x)}$

(see image on right)

Calcworkshop.com

$$\int 2(2x+1)^2 dx$$

$$u = 2x+1$$

$$du = 2 dx$$

$$\int (2x+1)^2 2 dx$$

$$= \int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{(2x+1)^3}{3} + C$$

genesso.edu

To the left is a worked problem showing how to integrate the function.

First, they picked their u. U was 2x+1 and they derived it to get du/dx=2.

Next, they plugged u in for 2x+1 and (du/2) in for dx. It is not seen because the two below the du cancelled with the 2 in the integral.

They then integrated in terms of u and plugged their x value for u back in.

Here is Baylor Tutoring YouTube video going over **U-Substitution!**

<https://www.youtube.com/watch?v=ANzdxIAivAQ>

<https://www.youtube.com/watch?v=-NZU-0j6FJ0>

https://www.youtube.com/watch?v=ipOVrYi_LTE

Highlight #1: Rules of U-Substitution

Remark [Some Common Integrals]:

i) Integral of a Constant: If k is a constant,

$$\int k \, dx = kx + C$$

ii) Integral of a Power Function: For $n \neq -1$,

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

iii) Integral of $\frac{1}{x}$:

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

iv) Integral of $\frac{f'(x)}{f(x)}$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

v) Integral of the Exponential Function:

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$$

Remark [Suggestions for Substitutions]: If the integrand involves

i) a rational function, $f(x) = \frac{g(x)}{h(x)}$, let $u = h(x)$.

ii) a quotient with a natural logarithm in numerator, $f(x) = \frac{\ln(h(x))}{g(x)}$,
let $u = \ln(h(x))$.

iii) an exponential equation, $e^{f(x)}$, let $u = f(x)$.

iv) a power or radical, $(f(x))^3$ or $\sqrt{f(x)}$, let $u = f(x)$.

Calculus textbook

Highlight #1: Finding C

Now, whenever we integrate, we always put +C on the end (I hope everyone does!). How do we get rid of it? Finding C is a common test problem, but we are actually able to solve for C! You must repeat the process of integrating all the way as shown above. Next, we will be given a point (x,y) and we can plug these into our equation for x, and f(x), respectively. The result leaves only one variable to solve for – C!

Function	Integral
$\rightarrow k$, a constant	$kx + c$
$\rightarrow x^n$	$\frac{x^{n+1}}{n+1} + c, n \neq -1$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c, x > 0$
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$

① Find $f(x)$ $f(x) = y$
 $f'(x) = 2x - 1$ and $f(0) = 3$

$$f(x) = \int 2x - 1 \, dx$$

$$f(x) = \frac{2x^2}{2} - 1x + C$$

$$f(x) = x^2 - x + C$$

$$3 = 0^2 - 0 + C$$

$$3 = C$$

$$f(x) = x^2 - x + 3$$

CHECK YOUR LEARNING

(Answers below at the end of the document.)

1) $f(x) = \int 2x(3x^2 + 3)^7 dx$

2) $f(x) = \int \frac{3x+6}{x^2+4x-3} dx$

3) $f(x) = \int (x + 1)^2 dx$ and the line passes through the points (2,11)

Things you might struggle with

U-Substitution: Substitution can be confusing because it is not always clear what to substitute but recognize the composite function and that inner function is what is used for "u".

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

ANSWERS to check your learning section

1. Answer: $F(x) = \frac{1}{24} (3x^2 + 3)^8 + C$
2. Answer: $F(x) = \frac{3}{2} \ln(x^2 + 4x - 3) + C$
3. Answer: $F(x) = \frac{(x+1)^3}{3} + 2$ (C=2)